Critical Values for Yen's Q_3: Identification of Local Dependence in the Rasch model using Residual Correlations

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UK Rasch Users Group Meeting:
18th March 2016, Durham

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Background: Local Dependence

The problem

Local independence of items is a fundamental assumption of the Rasch model

- The items should only be correlated through the latent trait that the test is measuring
- Item responses are conditionally independent given the latent variable

Background: Local Dependence

	True	Not True
I can walk a Kilometre on flat ground		
I can walk half a Kilometre on flat ground		
I can walk 100 metres on flat ground.		

Background: Local Dependence

The issue

- Relationship no longer probabilistic
- Artificial inflation of reliability estimates (Cronbach's alpha; Person Separation Index)
- Problems with construct validity/dimensionality

Local Dependence Detection

Residual Correlations

- Utilised within RUMM and WinSTEPS
- Yen's Q_3 (Yen, 1984)

Person-Item Residual Correlation Matrix

Principal Component Summary | PC Normalised Vectors | PC Loadings | Varimax Rotation Loadings | Residua

Item	I0001	I0002	I0003	I0004	I0005	I0006	I0007	I0008	I0009	I0010
I0001	1.000									
I0002	0.267	1.000								
I0003	-0.194	-0.249	1.000							
I0004	-0.056	-0.157	0.131	1.000						
I0005	-0.243	-0.272	0.020	-0.069	1.000					
I0006	-0.039	0.182	-0.209	-0.188	-0.174	1.000				
I0007	-0.089	-0.174	-0.157	-0.085	-0.154	-0.199	1.000			
I0008	-0.193	-0.173	-0.116	-0.143	-0.118	-0.202	0.062	1.000		
I0009	-0.093	0.015	-0.243	-0.266	-0.109	0.057	-0.196	-0.081	1.000	
I0010	0.022	-0.010	-0.189	-0.094	-0.199	-0.046	-0.034	-0.092	-0.017	1.000

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Local Dependence Detection

Residual Correlations

- When investigating LD based on Yen's Q_3, residuals for any pair of items should be uncorrelated, and generally close to 0.

- Residual correlations that are high indicate a violation of the local independence assumption, and this suggests that the pair of items have something more in common than the rest of the item set have in common with each other (Marais, 2013).

Local Dependence Detection

Issues with using Q3 Residual Correlations

- A negative bias is present in the residual correlations due to the way that they are calculated
- The sampling properties among residuals are unknown, so no formal tests of local dependency can be carried out
- No well-documented suggestions of the critical values which should be used to indicate LD, which has led to arbitrary rules-of-thumb being used

Local Dependence Detection

Residual Correlations

- At what level should these be taken to indicate dependency?
- Many examples in the literature of critical values ranging from 0.1-0.7.
- Should this be a straight cut-point?
- Should this be relative to the average correlation?

Local Dependence Detection

Marais (2013):

- Residual correlations are difficult to directly interpret confidently when there are fewer than 20 items in the item set.
- The magnitude of a residual correlation value which indicates LD will vary depending on the number of items in a data set.
- Correlations should always be considered relative to the overall set of correlations.

Study Aim

To attempt to find the empirical residual correlation critical value that should be applied to indicate LD

Methods

- Simulated data sets under the Rasch model, i.e. data sets without local dependence.
- Computed the empirical correlation matrix for each simulation
- Extracted the largest value from the correlation matrix

  = the empirical distribution of the **largest** dependency present under the condition of independence

Methods

- Simulations were carried out for all combinations of the four conditions:
  - Number of items (I = 10, 15, 20)
  - Number of persons (N =200, 250, … , 1000)
  - Number of response categories (two, four)
  - Targeting ('Good'/'Bad')

    (mean value of persons = 0, 2 logits)
- This yielded 204 different setups
- 10,000 data sets were simulated for each combination in order to find the empirical 95th and 99th percentiles

Results

Q_3Max = Maximum observed value within the residual correlation matrix

Q_{3*} = Q_3Max – the **average** residual correlation within the matrix

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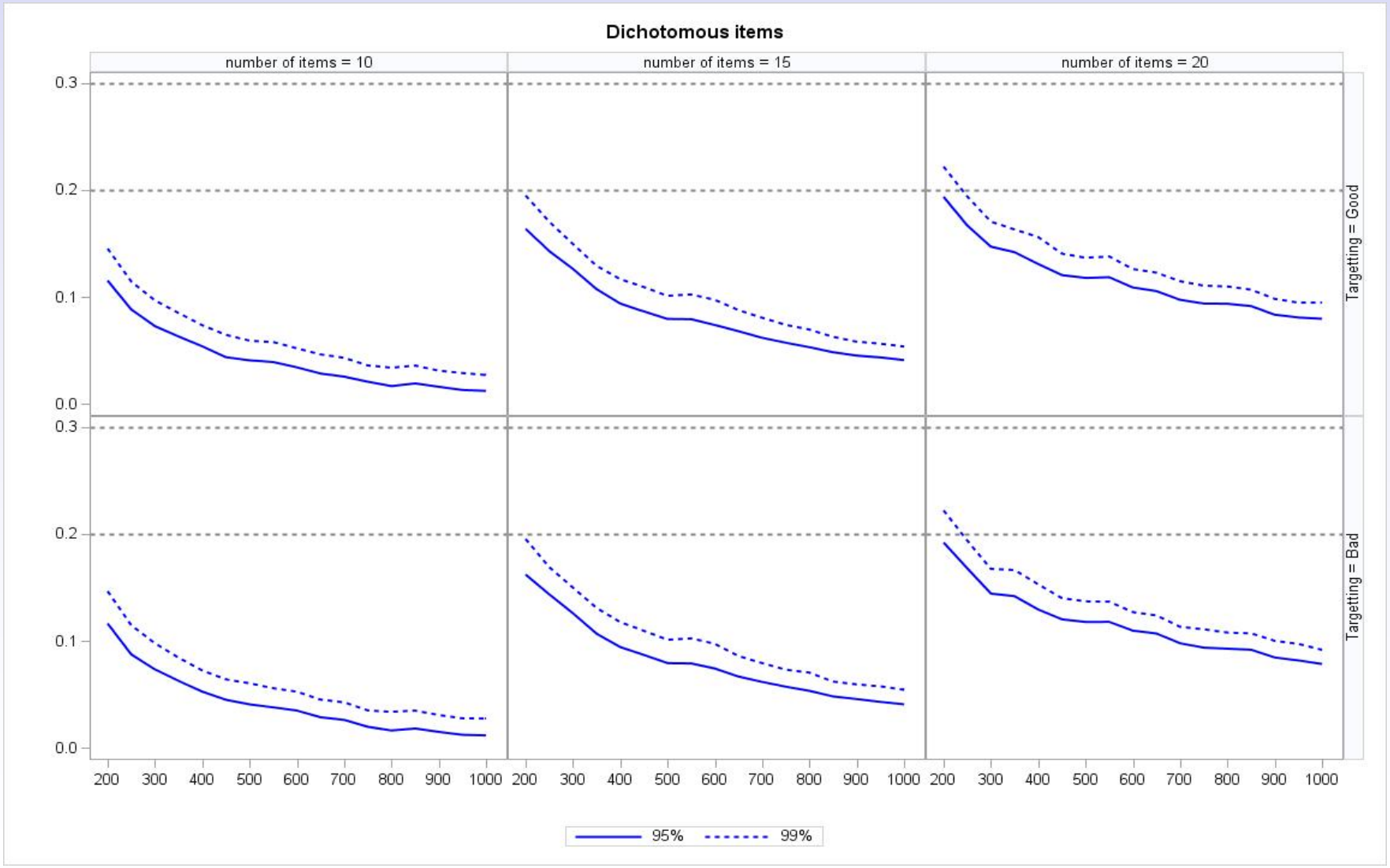
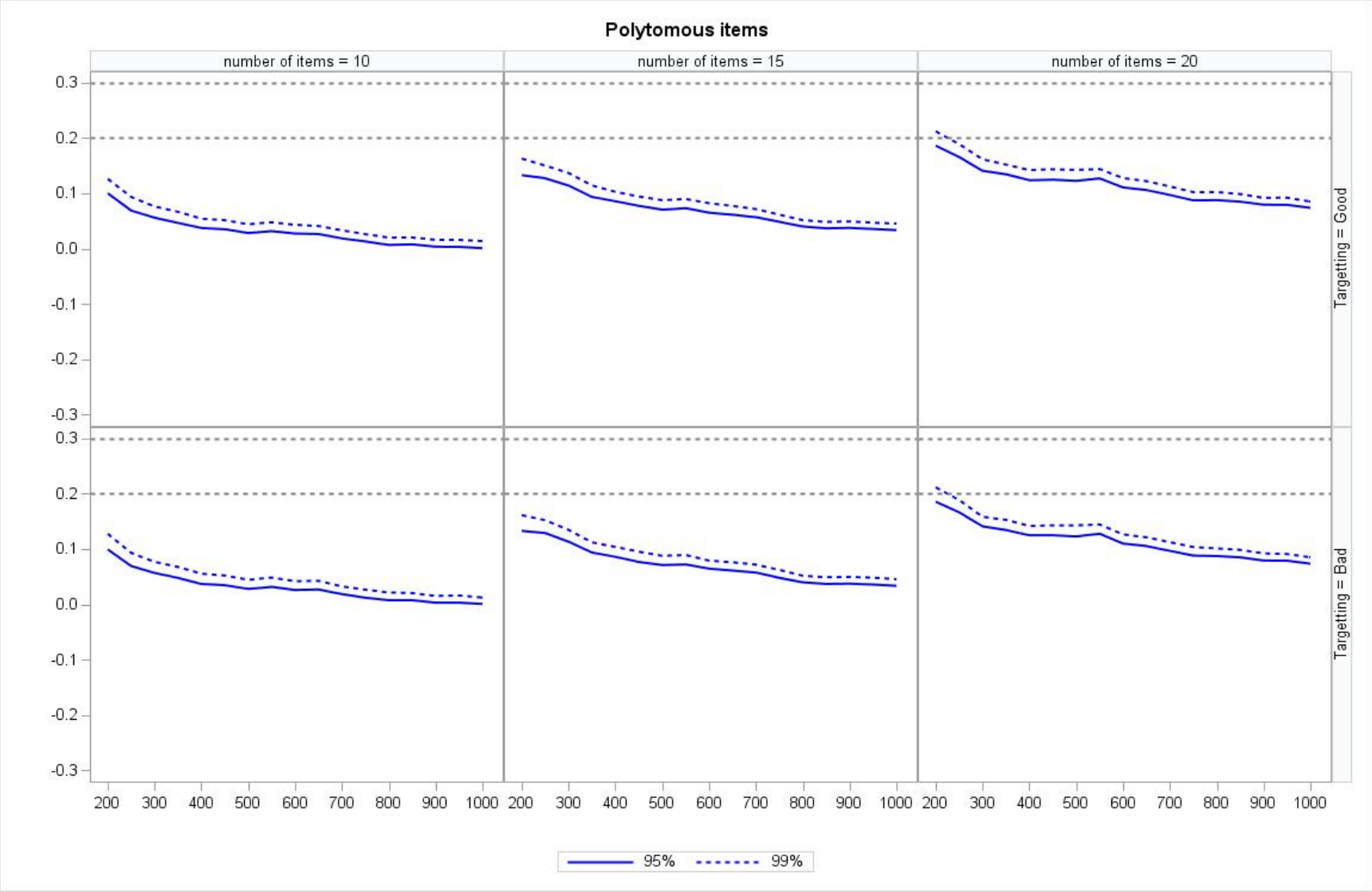
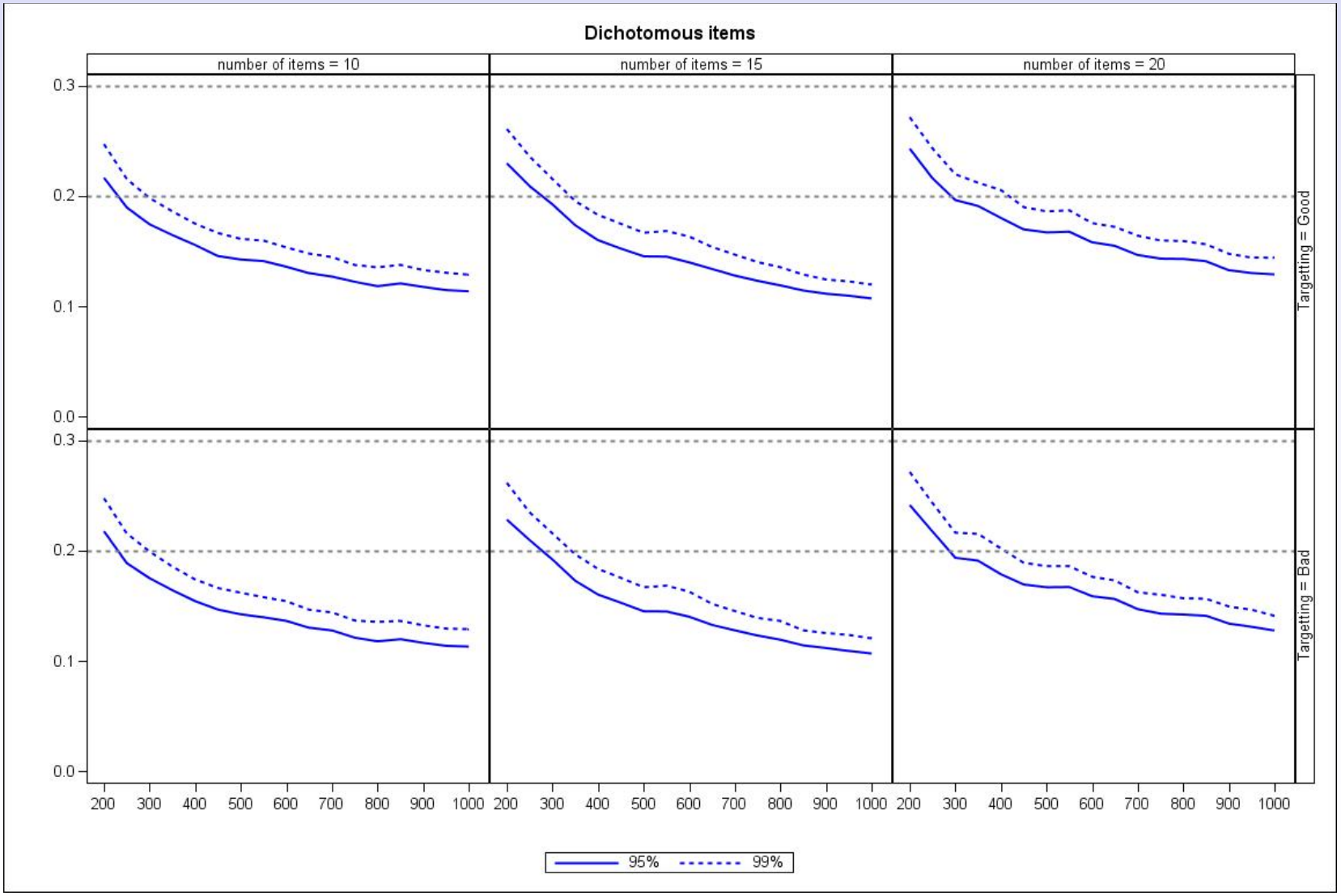
Dichotomous Results for Q_3Max



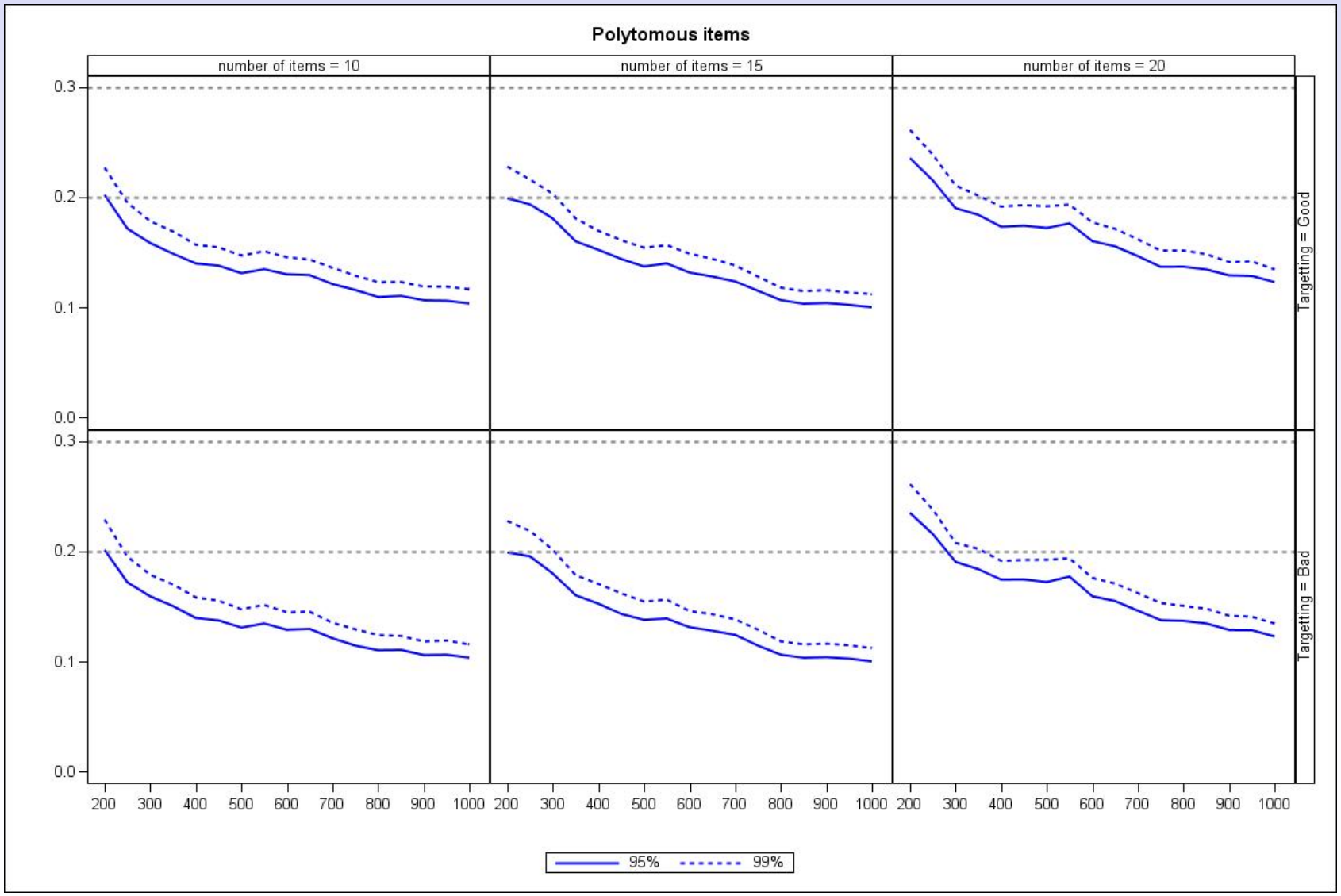
Polytomous Results for Q_3Max



Dichotomous Results for Q_{3*}



Polytomous Results for Q_{3*}



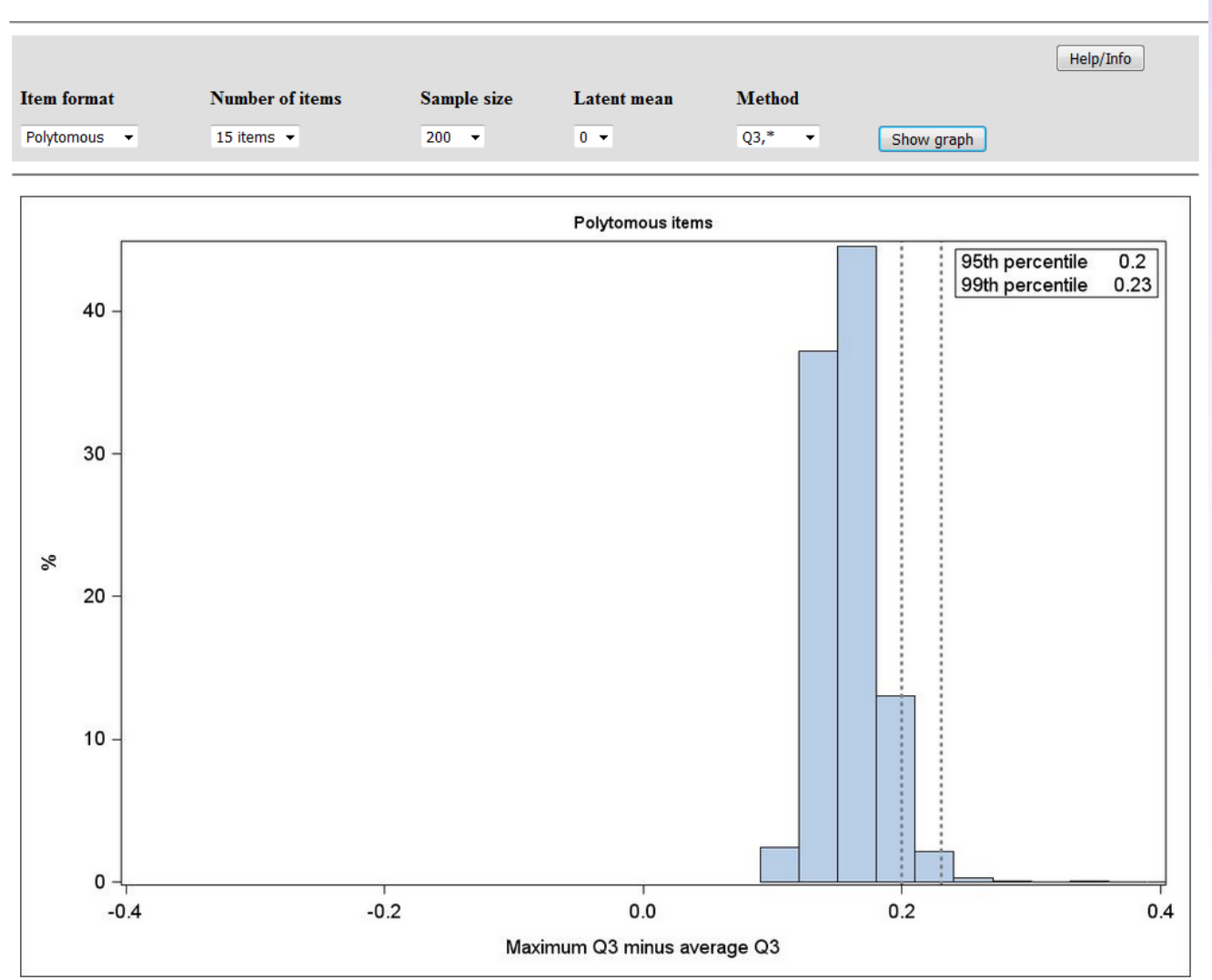
Conclusions

- No single critical Q_3 value is appropriate for all situations
- Empirical null distribution are influenced by:
  - number of items
  - sample size
  - number of response categories
- Targeting doesn't appear to have a big impact
- Local dependence should be considered relative to the average observed residual correlation

Q_3 Guidance Website

- http://publicifsv.sund.ku.dk/~kach/Q3/critical_values_Yens_Q3.html

Q_3 Guidance Website



Information

- Research report available at:

  https://ifsv.sund.ku.dk/biostat/annualreport/index.php/ResearchReport:RR-2015-No5

- Contact: m.c.horton@leeds.ac.uk

References:

Yen, W. M. (1984). Effects of Local Item Dependence on the Fit and Equating Performance of the Three-Parameter Logistic Model. Applied Psychological Measurement, 8(2), 125-145. doi:10.1177/014662168400800201

Marais, I. (2013). Local Dependence. In: Christensen KB, Kreiner S, Mesbah M (Eds). Rasch models in health. ISTE Ltd., London, UK.

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